



a review of Braided skew monoidal categories.by Bourke, John; Lack, Stephen

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Braided skew monoidal categories. (English) Zbl 07161907

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The notion of *skew monoidal category* was introduced in [*K. Szlachányi*, Adv. Math. 231, No. 3–4, 1694–1730 (2012; [Zbl 1283.18006](#))] in order to deal with bialgebroids. It is, roughly speaking, obtained from the familiar notion of *monoidal category* by loosening isomorphisms on associativity and unit conditions. This paper answer the question whether there exists a sensible notion of braiding for skew monoidal categories, generalizing the classical theory of braided monoidal categories [*A. Joyal* and *R. Street*, Adv. Math. 102, No. 1, 20–78 (1993; [Zbl 0817.18007](#))]. The authors give a notion of braiding on a skew monoidal category which is given by an invertible natural transformation

$$s : (AB)C \rightarrow (AC)B$$

pursuant to certain axioms. If B is a bialgebra, one gets a skew monoidal structure $\mathbf{Vect}[B]$ on the category \mathbf{Vect} of vector spaces with product

$$X \star Y = X \otimes B \otimes Y$$

and the ground field K as its unit. Bialgebroids give rise to, and can be characterized by certain skew monoidal categories [*K. Szlachányi*, Adv. Math. 231, No. 3–4, 1694–1730 (2012; [Zbl 1283.18006](#))].

It is shown in Theorem 4.10 that braidings on the skew monoidal category are in bijection with *cobraidings* (a.k.a. *coquasitriangular structures* [*C. Kassel*, Quantum groups. New York, NY: Springer-Verlag (1995; [Zbl 0808.17003](#)); *R. Street*, Quantum groups. A path to current algebra. Cambridge: Cambridge University Press (2007; [Zbl 1117.16031](#)))] on the bialgebra B . The theorem is presented as a specialized one of the main theorem Theorem 4.7 in §4 claiming that, given a monoidal comonad G on a monoidal category \mathcal{C} abiding by a mild hypothesis, there is a bijection between braidings on the monoidal category \mathcal{C}^G of coalgebras and braidings on the cowarped skew monoidal category $\mathcal{C}[G]$.

The other leading class of examples in the paper arises naturally if one attempts to study 2-categorical structures as strictly as possible, e.g., by considering the 2-category \mathbf{FProd}_s of categories endowed with a choice of finite products and functors which *strictly* preserve them in place of \mathbf{FProd} of categories with finite products and functors preserving finite products up to isomorphisms. The skew monoidal structure on \mathbf{FProd}_s is much easier to construct [*J. Bourke*, J. Homotopy Relat. Struct. 12, No. 1, 31–81 (2017; [Zbl 1417.18001](#))]. Indeed, they are exhibited in §6 after braided skew multicategories are introduced and it is shown how to pass from these to braided skew monoidal categories under the assumption of a representability condition in §5.

Reviewer: [Hirokazu Nishimura \(Tsukuba\)](#)

MSC:

- [18M50](#) Bimonoidal, skew-monoidal, duoidal categories
- [18M15](#) Braided monoidal categories and ribbon categories
- [18N10](#) 2-categories, bicategories, double categories
- [18N40](#) Homotopical algebra, Quillen model categories, derivators
- [16T10](#) Bialgebras

Keywords:

braiding; skew monoidal category; bialgebra; quasitriangular; 2-category

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